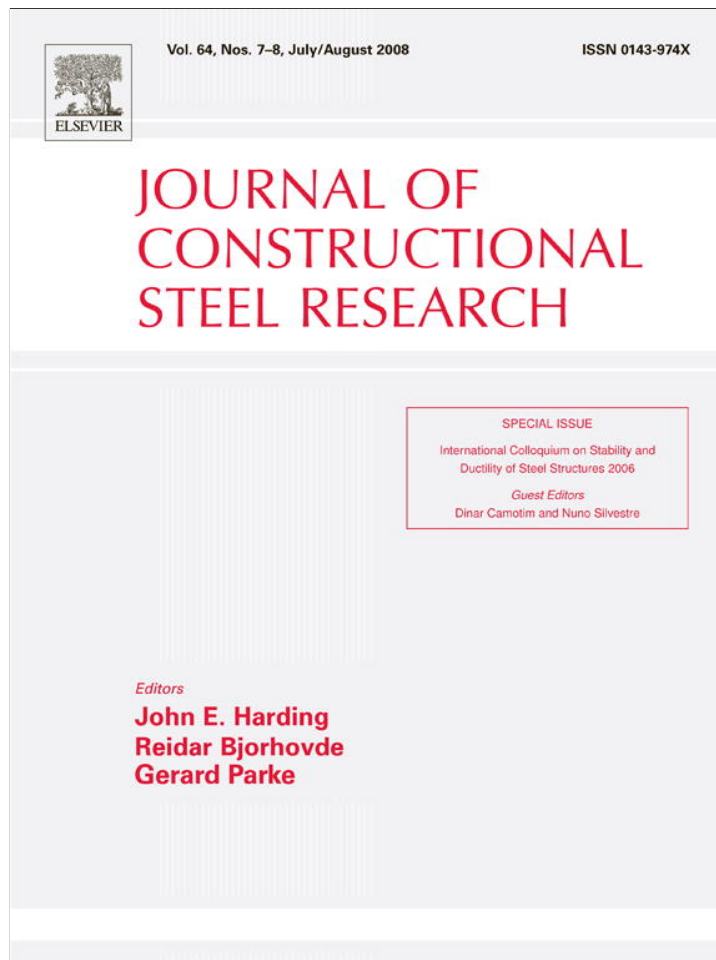


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# Review: The Direct Strength Method of cold-formed steel member design

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Received 30 April 2007; accepted 18 January 2008

## Abstract

The objective of this paper is to provide a review of the development and current progress in the Direct Strength Method for cold-formed steel member design. A brief comparison of the Direct Strength Method with the Effective Width Method is provided. The advantage of methods that integrate computational stability analysis into the design process, such as the Direct Strength Method, is highlighted. The development of the Direct Strength Method for beams and columns, including the reliability of the method is provided. Current and ongoing research to extend the Direct Strength Method is reviewed and complete references provided. The Direct Strength Method was formally adopted in North American cold-formed steel design specifications in 2004 as an alternative to the traditional Effective Width Method. The appendices of this paper provide the Direct Strength Method equations for the design of columns and beams as developed by the author and adopted in the North American Specification.

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*Keywords:* Direct Strength Method; Effective Width Method; Cold-formed steel; Stability; Finite strip method; Thin-walled

## 1. Introduction

Cold-formed steel members are typically thin-walled, i.e. local plate buckling and cross-section distortion must be treated as an essential part of member design. These complications also provide certain opportunities, as local plate buckling, in particular, has the capacity for beneficial post-buckling reserve that can be drawn upon for increased strength in design. As a result, the ultimate efficiency, e.g. in terms of strength-to-weight ratio, can be quite high for cold-formed steel members. The challenge for any cold-formed steel design method is to incorporate as many of these complicated phenomena, that are largely ignored in conventional design of ‘compact’ sections, into as simple and familiar a design method as possible. Further complicating the creation of simple design methods for cold-formed steel members is the lack of symmetry in many cross-sections, the enhanced possibility of limit states related directly to the use of thin steel sheet such as web crippling, and other unique characteristics of their manufacture and application.

## 2. Design methods for thin-walled members

Currently, two basic design methods for cold-formed steel members are formally available in design specifications in

North America the traditional Effective Width Method, also known as the unified method or the main specification method [1], and the Direct Strength Method, also known as the Appendix 1 method [2]. The Effective Width Method is available, in some form, nearly world-wide for formal use in design, while the Direct Strength Method has only been adopted in North America, and Australia/New Zealand. Other design approaches include: Reduced Stress, Effective Thickness, the Q-factor approach and more recently the Erosion of Critical Bifurcation Load approach championed by Dubina [3,4], all of which are worthy of mention, but not detailed here further.

### 2.1. Effective Width Method

The basis for the Effective Width Method is well explained in textbooks and Specifications; the essential idea is that local plate buckling leads to reductions in the effectiveness of the plates that comprise a cross-section, as demonstrated in Fig. 1(a). More formally, this loss in plate effectiveness can be understood as an approximate means to account for equilibrium in an effective plate under a simplified stress distribution as opposed to the actual (full) plate with the actual nonlinear longitudinal stress distribution that develops due to buckling, as illustrated in Fig. 1(a). Each plate in a cross-section is reduced

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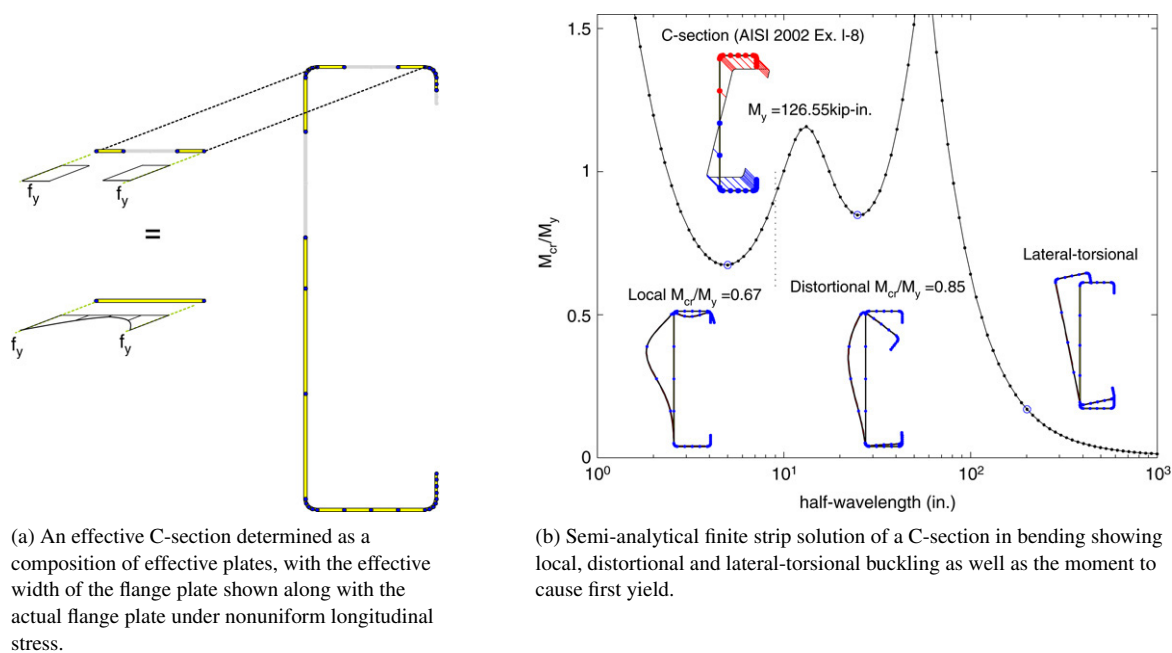


Fig. 1. Fundamental steps in the strength determination of a C-section by (a) Effective Width Method and (b) Direct Strength Method.

to its effective width, and this reduction from the gross cross-section to the effective cross-section, again as illustrated in Fig. 1(a), is fundamental to the application of the Effective Width Method. The effective cross-section (i) provides a clear model for the locations in the cross-section where material is ineffective in carrying load, (ii) cleanly leads to the notion of neutral axis shift in the section due to local-buckling and (iii) provides an obvious means to incorporate local–global interaction where reduced cross-section properties influence global buckling (although specifications often simplify this interaction somewhat).

However, the common two-dimensional nonlinear stress distribution that is shown to explain the effective width of a plate is itself an approximation, representing the average of the longitudinal membrane stress and ignoring variation in stress through the thickness as well as variation in stress along the length of the plate. Thus, the true “effective width” is far more complicated than typically assumed and existing effective width equations only correlate to average membrane stress conditions in a plate. Further, the Effective Width Method (i) ignores inter-element (e.g. between the flange and the web) equilibrium and compatibility in determining the elastic buckling behaviour, (ii) incorporation of competing buckling modes, such as distortional buckling can be awkward, (iii) cumbersome iterations are required to determine even basic member strength and (iv) determining the effective section becomes increasingly more complicated as attempts to optimize the section are made, e.g. folded-in stiffeners add to the plates which comprise the section and all plates must be investigated as being potentially partially effective. The Effective Width Method is a useful design model, but it is intimately tied to classical plate stability, and, in general, creates a design methodology that is different enough from conventional (hot-

rolled) steel design that it may impede use of the material by some engineers in some situations.

## 2.2. Direct Strength Method

If the effective width (or section) is the fundamental concept behind the Effective Width Method, then accurate member elastic stability, as shown in Fig. 1(b) is the fundamental idea behind the Direct Strength Method. The Direct Strength Method is predicated upon the idea that if an engineer determines all of the elastic instabilities for the gross section, i.e. local ( $M_{crl}$ ), distortional ( $M_{crd}$ ), and global buckling ( $M_{cre}$ ), and also determines the moment (or load) that causes the section to yield ( $M_y$ ), then the strength can be directly determined, i.e.  $M_n = f(M_{crl}, M_{crd}, M_{cre}, M_y)$ . The Direct Strength Method has been mentioned in textbooks and review articles [5–8]. The method is essentially an extension of the use of column curves for global buckling, but with application to local and distortional buckling instabilities and appropriate consideration of post-buckling reserve and interaction in these modes. The development of, and continued research into, the Direct Strength Method is explored further in this paper.

## 2.3. Long-term goals

It is important to recognize in any discussion regarding the Effective Width Method, the Direct Strength Method, or other semi-empirical design methods that none of these design methods are theoretically correct. Rather, a complicated nonlinear problem is simplified in some manner so that engineers may have a working model to design from without resorting to testing every individual member. These models serve us well when backed up by the application of reliability to incorporate uncertainty in their predictive powers.

It is this author's contention that the long-term goal for thin-walled member design should be a fully nonlinear computational simulation. To this end, the computational member elastic buckling stability analysis that is at the heart of the Direct Strength Method is a useful stepping stone. In particular, the underlying mechanics for the member stability solutions, in e.g. the finite strip method [9,10] are necessary (but not sufficient) for understanding fully nonlinear analysis. Such a nonlinear analysis will also need to incorporate geometrical and material imperfections into a consistent reliability framework so that we may provide engineers with a realistic prediction of strength and sensitivity that can be used in design.

More attempts to understand the inputs to thin-walled member strength such as geometric imperfections and residual stresses [11] as well as modelling assumptions (elements, material modelling) related to the underlying mechanics are needed [12]. Finally, for a full structural simulation the member analysis will need to be wedded to realistic connection and system models. While these remain long-term goals this author contends that we should try to place as much emphasis on mechanics that we can agree on today (such as member elastic stability) into current design codes and specifications — as we drive towards more robust solutions in the future.

### 3. Direct Strength Method for columns: Development

For columns, the beginning of the Direct Strength Method, though it was not called this at the time, can most clearly be traced to research into distortional buckling of rack post sections at the University of Sydney [13,14]. In particular, Hancock et al. [15] collected the research and demonstrated that for a large variety of cross-sections the measured compressive strength in a distortional failure correlated well with the slenderness in the elastic distortional mode. As is often the case with attempts to determine an origin, we can go back even further as Hancock attributes his methodology to Trahair's work on the strength prediction of columns undergoing flexural-torsional buckling. In this regard it becomes clear that the Direct Strength Method is not a new idea, but rather the extension of an old one to new instability limit states.

Development of the Direct Strength Method beyond distortional buckling was completed using a much wider set of cold-formed steel cross-sections and tests that included failures in local, distortional, and global flexural or flexural-torsional modes [16,17]. For the 187 columns gathered in [16,17] hand solutions and numerical (finite strip) solutions for the elastic buckling were calculated. For local buckling the strength curve was selected to be similar to that previously found for beams (see the following section for further discussion). For distortional buckling, one of the curves suggested by Hancock et al. in [15] was employed. For global buckling the existing specification expressions [1] were employed.

The resulting Direct Strength Method provisions for columns are summarized in the Appendix A of this paper and comparison with the test data is provided graphically in Fig. 2. Note, that for the local failures the normalization of

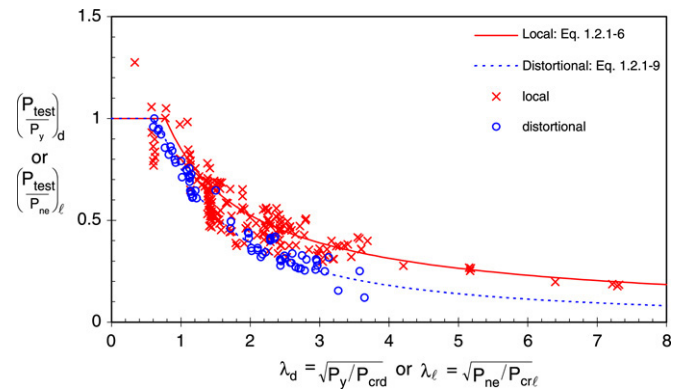


Fig. 2. Comparison of the Direct Strength Method predictor curves with test data for columns (equation numbers refer to those used in the North American Specification [2]).

$P_{test}$  is to  $P_{ne}$ , the maximum strength due to global buckling (thus reflecting local–global interaction), while for distortional buckling the normalization of  $P_{test}$  is to  $P_y$ , the squash load of the column. Fig. 2 indicates that the Direct Strength Method is a reasonable predictor of strength over a wide range of slenderness. Reliability of the method is further discussed in Section 5.

Interaction of the buckling modes was systematically studied for local–global, distortional–global, and local–distortional buckling of the columns. Based on overall test-to-predicted ratios, and when available the failure modes noted by the researchers in their testing, it was determined that local–global interaction should be included, but not distortional–global, or local–distortional interaction. For instance, if local–distortional interaction is included, by replacing the maximum load in the Direct Strength Method provisions with the distortional strength,  $P_{nd}$ , instead of the global strength,  $P_{ne}$ , (see Appendices A and B for the expressions) this results in overly conservative predictions: 169 of the 187 tests would be identified to fail in local–distortional interaction and the average test-to-predicted ratio would be 1.35 [16,17]. Neither the failure mode nor strength prediction is consistent with the observations from the tests when local–distortional interaction is included for all columns. As a result, it was recommended to only include local–global interaction in the Direct Strength Method.

Recent work [18,19] has questioned whether local–distortional interaction should be included in some specific cases, particularly when the elastic critical local and distortional buckling loads are at similar levels. Work is ongoing to determine the most appropriate way to identify and predict the strength for the small number of columns that do have potential local–distortional interaction.

### 4. Direct Strength Method for beams: Development

The first mention of the Direct Strength Method occurs in [20] and was closely coupled to the development of the method for beams, in particular, application of the large database of sections that was collected by the author to explore two problems: distortional buckling in C- and Z-section beams, and local and distortional buckling in deck sections with

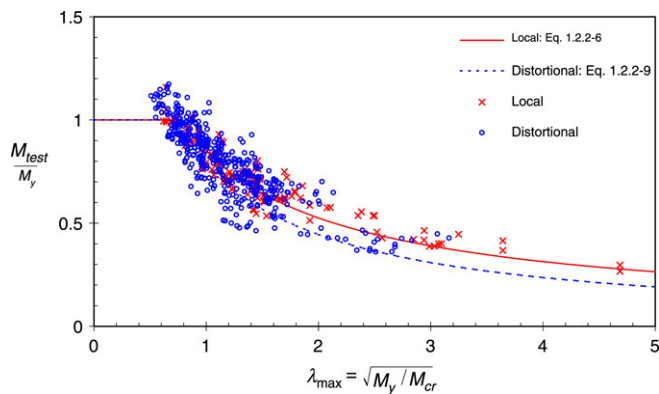


Fig. 3. Comparison of the Direct Strength Method predictor curves with test data for beams (equation numbers refer to those use in the North American Specification [2]).

multiple longitudinal intermediate stiffeners in the compression flange. At the same time Hancock and related researchers at the University of Sydney demonstrated that distortional buckling failures for a wide variety of failures were well correlated with the elastic distortional slenderness [15,21]. The form of the presentation of the Direct Strength Method for beams evolved somewhat from [20]. In particular, curve (2) of [20] as discussed in [22] is identical to the distortional buckling expressions developed in [21] and became the distortional buckling Direct Strength curve. For local buckling, curve (3) of [20] was employed. Appendix B of this paper provides the Direct Strength Method expressions for beams, and the performance against experimental data is graphically provided in Fig. 3.

Note, for the beam data of Fig. 3 all of the  $M_{test}$  values are normalized against the moment at first yield,  $M_y$ . This is due to the fact that all of the test data employed were for laterally braced members. It is worth noting that while local–global interaction was experimentally examined for columns, and the same methodology applied for beams, local–global, distortional–global, and local–distortional interactions have not been experimentally examined in the context of the Direct Strength Method for beams. Based on the findings for columns local–global interaction has been included and local–distortional and distortional–global interactions ignored. The performance of laterally unbraced beams deserves further study, not only in the context of the Direct Strength Method and potential interactions, but also to better understand how warping torsion should be treated. For moderate rotations the influence of the torsional stress on local and distortional buckling modes is real [23] and its potential inclusion in the Direct Strength Method is worthy of further study.

The beam data of Fig. 3 show far more distortional buckling failures than the column data of Fig. 2. This is due to two reasons: (i) distortional buckling failures are more common in typical C- and Z-sections where the web is stabilized by the tensile portion of the bending stress and (ii) the database of sections includes a large number of deck and hat sections with longitudinal intermediate stiffener(s) in the compression flange — buckling of those members in which the longitudinal

intermediate stiffeners are engaged is defined as distortional buckling failures.

In the development of the Direct Strength Method for C- and Z-section beams separation of local and distortional buckling failure modes was initially somewhat difficult and complicated by the bracing and boundary conditions used in the testing, which typically restrained distortional buckling in part, but not necessarily in full. Nonetheless, expressions were arrived at as provided in the Appendices A and B and adopted in [2]. A recent series of flexural tests and complementary finite element analysis on a variety of C- and Z-sections in local buckling [24–26] and distortional buckling [25–27] used specific details to isolate the two modes and unequivocally demonstrated the robustness of the Direct Strength Method predictions for C- and Z-sections failing in either the local or distortional mode. A summary of the performance of these sections is provided in Fig. 4. Recently, additional testing focused on distortional buckling has also been completed [28].

Finally, it is worth noting that the testing on C- and Z-section beams has focused on strong-axis bending and associated buckling, extension to weak-axis bending has been assumed. This assumption is justified in part by the inclusion of hats and decks in the experimental database, these sections are bent about their weak-axis, and are similar in their behaviour to a C-section in weak axis bending. Further, the major-axis bending modes are considered more critical since the primary effect of weak-axis bending in comparison to strong-axis bending is the elimination of global lateral-torsional buckling modes.

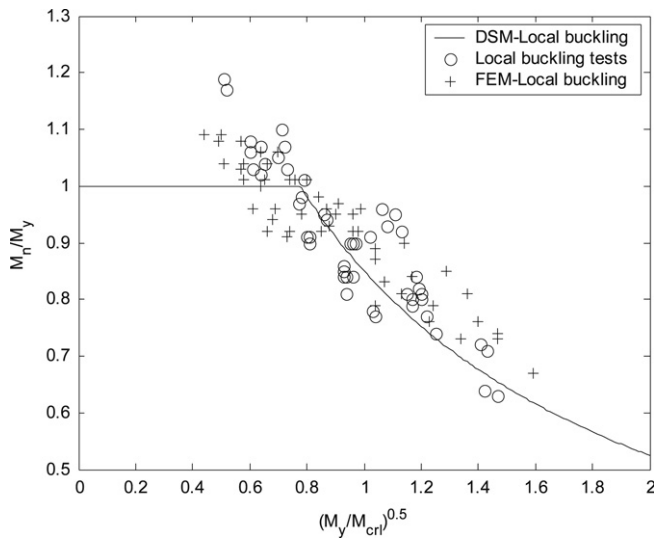
## 5. Reliability and comparison to Effective Width Method

### 5.1. Reliability

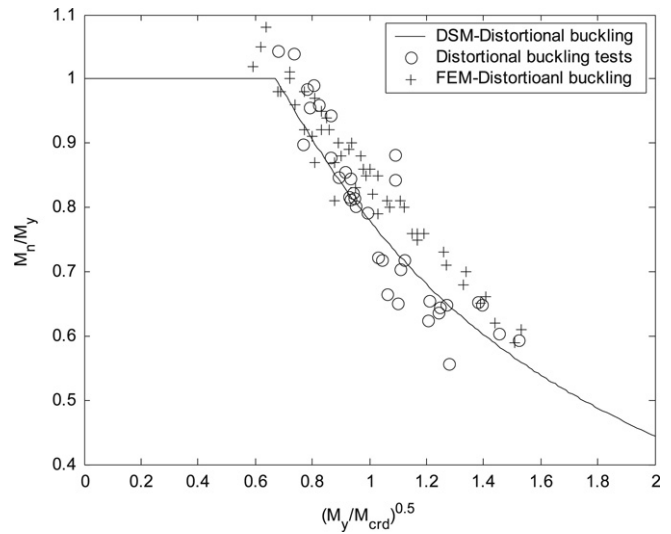
The reliability of the Direct Strength Method was established using the limit-states design format in use in the United States: Load and Resistance Factor Design (LRFD). Chapter F of [29] provides the formal expressions for deriving the resistance factors,  $\phi$  where capacity ( $\phi R_n$ ) must be greater than demand ( $\gamma Q$ ) in the LRFD format via  $\phi R_n > \sum \gamma_i Q_i$ . A target reliability  $\beta$  of 2.5 was employed. The resulting resistance factors ( $\phi$ 's) for the Direct Strength Method of [2] and the Effective Width Method of [1] are provided in Table 1.

Overall, Table 1 indicates that the reliability of the Direct Strength Method is as good, or better than then Effective Width Method. Table 1 also indicates that for beams the Effective Width Method has a lower reliability than the target reliability (calculated  $\phi$  is less than specified  $\phi$ ), this is attributed primarily to the inability of [1] to properly account for the distortional buckling limit state. For the Direct Strength Method, most noticeable is the approximation inherent in using a single  $\phi$  factor for beams (or columns), instead of different  $\phi$  factors for each limit state, i.e. local or distortional. The decision to use a single  $\phi$  factor introduces a certain level of approximation in the method.

For the Direct Strength Method the statistics used in the reliability calculation of Table 1 are summarized in Table 2;



(a) Local buckling in beams.



(b) Distortional buckling in beams.

Fig. 4. Comparison of Direct Strength Method for beams to tests and additional FE results for C and Z sections in (a) local and (b) distortional buckling.

Table 1  
Reliability of design methods

	$\phi$	
	Beams	Columns
AISI (1996) specification [1]		
Based on DSM data <sup>a,b</sup>	0.77	0.82
Specified	0.90 or 0.95	0.85
Direct Strength Method [2]		
Local ( $M_{n\lambda}$ or $P_{n\lambda}$ controls)	0.89	0.79
Distortional ( $M_{nd}$ or $P_{nd}$ controls)	0.93	0.90
Combined	0.92	0.85
Specified in [2]	0.90	0.85

<sup>a</sup> Sections which are outside the geometrical bounds of [1] or include longitudinal web stiffeners or other features not covered in [1] are excluded from the calculation.

<sup>b</sup> The DSM data includes all the tested sections cited in [2] as shown in Figs. 2 and 4.

Table 2  
Summary statistics for Direct Strength Method development

	$n$	$P_m$	$V_p$
Beams			
C-sections	185	1.10	0.11
C-sections with web stiffeners	42	1.12	0.07
Z-sections	48	1.13	0.13
Hat sections	186	1.10	0.15
Trapezoidal sections	98	1.01	0.13
ALL BEAMS	559	1.09	0.12
Columns			
C-sections	114	1.01	0.15
C-sections with web stiffeners <sup>a</sup>	29	0.88	0.14
Z-sections	85	0.96	0.13
Rack sections	17	1.02	0.05
Hat sections	4	0.98	0.02
ALL COLUMNS	249	0.98	0.14

<sup>a</sup> Thomasson's (1978) tests contribute to the low  $P_m$ , more recent tests by Kwon Hancock (1992) showed much better agreement. See [2] or [37] for full citations and further details.

included are the sample size,  $n$ , mean test-to-predicted ratio,  $P_m$ , and coefficient of variation,  $V_p$ , broken down by use (beam or column) and cross-section type. Table 2 underscores the relatively large sample size of tests used to develop the Direct Strength Method and the overall statistical accuracy of the approach. Some statistical bias based on the cross-section type is observed; this bias is ignored in the current implementation of the method.

5.2. Element interaction

While the reliability calculations provides an overall comparison of the Effective Width Method and the Direct Strength Method they do not shed much light on the detailed differences between the two methods. For example, for columns the Effective Width Method and the Direct Strength Method provide similar levels of overall reliability, but do so in very different ways. Systematic error in the strength prediction of columns using the Effective Width Method [1] is demonstrated in Fig. 5. Recent work [30–33] has underscored the importance of sharing these more detailed comparisons.

In Fig. 5 the strength predictions of the Effective Width Method and the Direct Strength Method are compared as a function of the web slenderness of a C-section column. As web slenderness increases the Effective Width Method solution becomes systematically unconservative. This behaviour is exacerbated by the fact that for typically available C-sections as the web becomes deeper the flange width remains at approximately the same width, so high web slenderness is strongly correlated with high web-to-flange width ratios (i.e. C-sections which are 'narrow'). This detrimental behaviour is primarily one of local web/flange interaction, not distortional buckling. Since the Effective Width Method uses an element approach, no matter how high the slenderness of the web becomes, it has no effect on the solution for the flange. In contrast the Direct Strength Method of Fig. 5(b), which includes element interaction (i.e. interaction between the flange

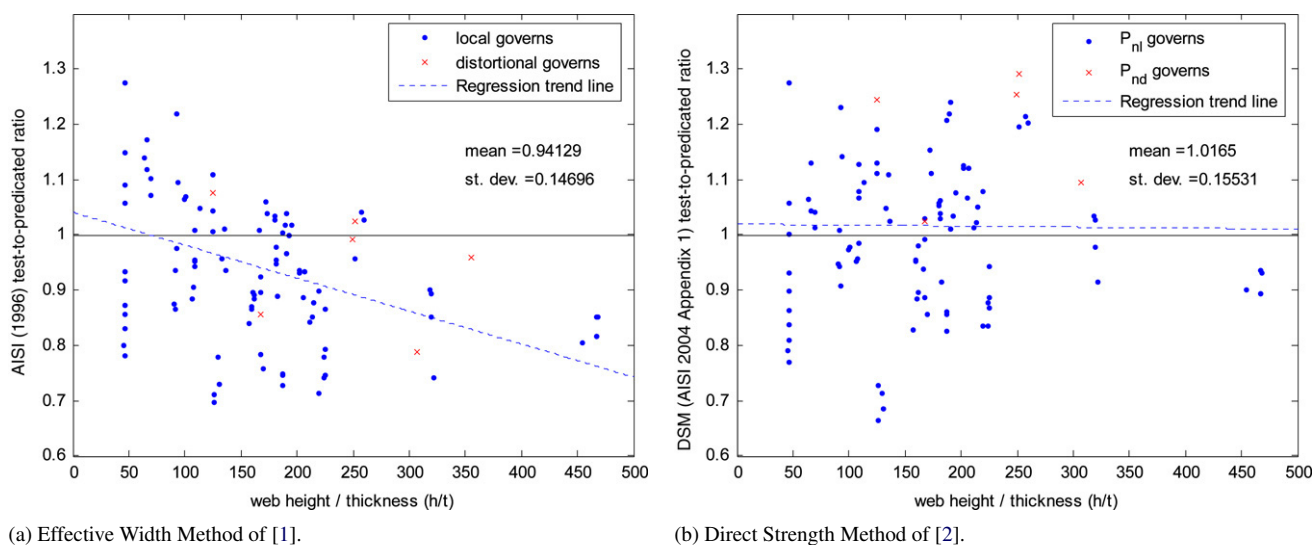


Fig. 5. Test-to-predicted ratio for (a) the Effective Width Method of [1] and (b) the Direct Strength Method of [2] for all lipped channel columns used in the development of Direct Strength Method predictor equations plotted as a function of web slenderness ( $h/t$ ).

and the web), performs accurately over the full range of web slenderness. Proper inclusion of element interaction is necessary for accurate strength prediction of these columns.

Taken to extremes, inclusion of elastic element interaction can also work against the Direct Strength Method, making the method overly conservative. This fundamental limitation of the Direct Strength Method was reported in the first paper to propose the approach [20]. When one part (element) of the cross-section becomes extraordinarily slender that element will drive the member elastic critical buckling stress to approach zero. The Direct Strength Method will assume the member strength, like the member elastic critical buckling stress, will also approach zero. In contrast, the Effective Width Method presumes only that the element itself (not the member) will have no strength in such a situation. Deck or hat sections in bending with low yield stress and very slender (wide) compression flanges without intermediate stiffeners tend to fall in this category and thus have unduly conservative predictions by the Direct Strength Method, but quite reasonable predictions via the Effective Width Method. However, ignoring inter-element interaction, as the Effective Width Method traditionally does, is not universally a good idea as illustrated for the C-section columns in Fig. 5.

For optimized deck sections with multiple longitudinal intermediate stiffeners in the web and the flange (see e.g. [34]) the Direct Strength Method is highly desirable over the Effective Width Method — here the benefit is primarily convenience not theoretical. If a computational solution is employed for determining the elastic buckling stresses (moments) an optimized deck section is no more complicated than a simple hat for strength determination; but for the Effective Width Method the calculation of effective section properties and accurately handling the effective width of the numerous sub-elements leads to severe complication without increased accuracy, or worse in the case of many specifications (e.g. [1] or [29]) no design approach is even available for such a section using the Effective Width Method. In general, as

sections are optimized the Direct Strength Method provides a simpler design methodology with wider applicability than the Effective Width Method.

## 6. Practical developments

Implementation of the Direct Strength Method has required a number of practical developments beyond the initial research. This section covers these practical developments as related to the Direct Strength Method adopted in [2]. These developments focus on three main areas: the definition and use of prequalified sections, performing serviceability (deflection) calculations using the Direct Strength Method, and design aids developed for engineers employing the Direct Strength Method in practice.

### 6.1. Prequalified sections

During the formal codification of the Direct Strength Method in [2] it was determined that the users of the method should be aware of the cross-sections employed to verify the approach. Further, it was decided that the geometrical and material bounds of the cross-sections used in the verification of the Direct Strength approach should be able to use the derived  $\phi$  factors (Table 1), but new sections falling outside the boundaries of tested sections should use slightly reduced (more conservative)  $\phi$  factors. Thus, the idea of prequalified section (or limits) was established, and [2] includes a number of tables that provide the geometrical and material bounds for prequalified members. Essentially, the prequalified sections in [2] represent a summary of the experimental database used in verifying the Direct Strength Method. It is perhaps worthy to note that this experimental database is larger than that used for determining the Effective Width Method approach of [1] or [29].

6.2. Members with complex stiffeners and extension of prequalified sections

In 2006, based on the work in [35] and [36] the limits on pre-qualified sections in [2] were extended to cover C- and Z-section beams with complex lip stiffeners. For columns the category of Lipped C-Section and Rack Upright were merged, as a rack upright is a C-section with a complex stiffener. In addition, the complex stiffener limits from the original Rack Upright category were relaxed to match those found for C-section beams with complex stiffeners. Finally, the Effective Width Method of [29], i.e. the main Specification for North America, was restricted to only cover with simple lip stiffeners — thus the Direct Strength Method became the preferred approach for these more complicated sections.

6.3. Development of new and optimal cross-sections

No definitive method has yet been established for extending the limits of a prequalified section, but in [37] initial guidance is provided. Of particular interest is the potential to use a small number of tests and extend one of the prequalified categories — to this end the statistics of Table 2 ( $n, P_m, V_p$ ) are provided for use. For a new section the reliability may be calculated independently using Chapter F of [1] in the same manner as completed for Table 1. For a new section, which is similar to an existing section in most respects, the existing results ( $n, P_m, V_p$ ) may be combined with the new test results to determine if the new sections provide the same level of reliability as the old. Further details are provided in [37].

6.4. Deflection calculation (serviceability)

To examine serviceability, deflections are typically determined at the service stress level of interest. In the Effective Width Method, to account for reduced stiffness due to cross-section instability, the effective member properties are determined at the service stress. The Direct Strength Method uses a similar philosophy, but since the equations are in terms of strength, the implementation is more awkward. As detailed in [2] the service level moment ( $M$ ) is used as the peak moment (i.e.  $M$  replaces the yield moment  $M_y$  in the expressions) and the deflection strength  $M_d$  of the cross-section is determined. The ratio of these two moments ( $M_d/M$ ) provides an approximate reduction in the stiffness of the member at the service moment,  $M$ . Results of the calculation for a typical C-section are shown in Fig. 6 while the full solution is detailed in [37]. The approach follows the same basic trends as the Effective Width Method for reduced stiffness in a cross-section.

6.5. Design aids

As detailed in [38] a Design Guide for the Direct Strength Method [37] has recently been authored to aid engineers in the application of the Direct Strength Method. The Guide covers the following areas: elastic buckling, overcoming difficulties with elastic buckling determination in the finite strip method, beam design, column design, beam–column design and product

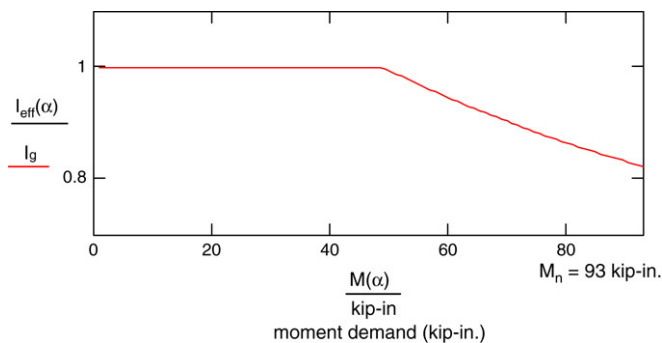


Fig. 6. Reduced stiffness as a function of service moment for a 9CS2.5x059.

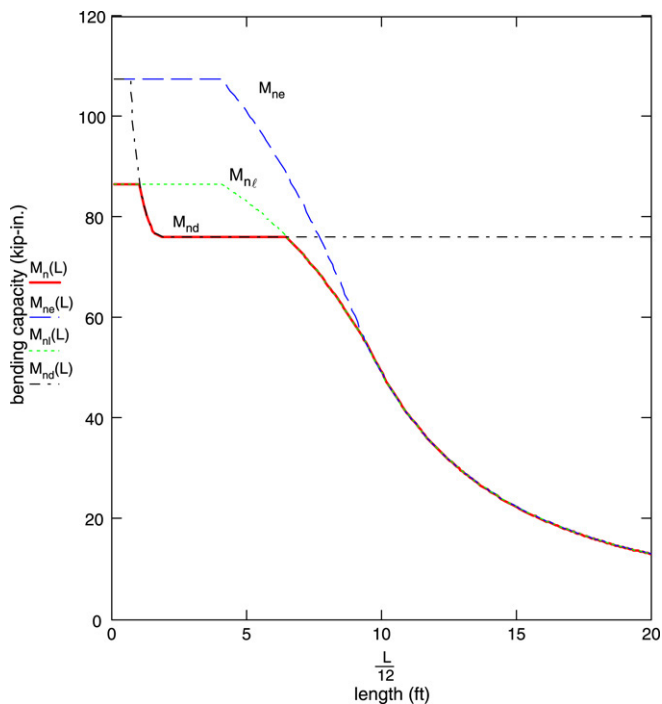


Fig. 7. Example beam chart for a Z-section with lips.

development. The Design Guide includes nearly 100 pages of design examples. The Design Guide provides a complete discussion of the details associated with application of the finite strip method, and the difficulties encountered as well. Topics covered include: indistinct local mode, indistinct distortional mode, multiple local or distortional modes (stiffeners), global modes at short unbraced lengths, global modes with different bracing conditions, influence of moment gradient, partially restrained modes, boundary conditions for repeated members, members with holes, boundary conditions at the supports not pinned, and built-up cross-sections. The discussion is directed at aiding engineers who need the finite strip method for more than just cursory use.

The Design Guide provide complete details for development of beam span tables or charts and column height tables or charts using the Direct Strength Method. An example beam chart is provided in Fig. 7. In this example one can readily see how the local buckling strength,  $M_{nl}$ , is a reduction below the global buckling strength,  $M_{ne}$ . The point where  $M_{nl}$  and  $M_{ne}$  merge (approximately 9 ft) indicates that local buckling



no longer provides a reduction in the strength of this beam (in the main Specification [29] this occurs when the stress used to determine the effective section,  $F_n$  is low enough that the section is fully effective at that stress.) Further, the detrimental impact of distortional buckling on intermediate length beams is shown in Fig. 7.

Additional information on the design of purlins using the Direct Strength Method beyond that in [27] is also offered in [39]. Further, the behavior of purlins as struts was explored in [40]; however, comparisons to the Direct Strength Method did not incorporate the beneficial influence of rotational restraint to the purlins as discussed in [37] and detailed in [25]. Built-up sections are explored in [37] and in recent research [41]. The work reported in [41] has been corrected since its publication and the authors should be contacted for corrected comparisons to the Direct Strength Method.

As engineers employ the Direct Strength Method on novel cross-sections one important piece of advice from [37] is that when in doubt about whether to define a given buckling mode as local or distortional it is always conservative to assume it is both modes. Such an approach is conservative, but ensures reduced post-buckling strength at intermediate unbraced lengths (i.e. the distortional reduction) as well as inclusion of interaction effects (i.e. local–global interaction).

## 7. Advancing the Direct Strength Method

A significant amount of research work is ongoing in relation to the Direct Strength Method. The following sections summarize recent research on the Direct Strength Method, most of the work detailed below has not yet been adopted in the Specification.

### 7.1. Shear

No formal provisions for shear currently exist for the Direct Strength Method. However, it is recommended in [37] that existing provisions [29] could be suitably modified. As a rational analysis extension the existing equations from [29] are recast into the Direct Strength format and are suggested for use

$$\text{for } \lambda_v \leq 0.815 \quad V_n = V_y \quad (1)$$

$$\text{for } 0.815 < \lambda_v \leq 1.231 \quad V_n = 0.815\sqrt{V_{cr}V_y} \quad (2)$$

$$\text{for } \lambda_v > 1.231 \quad V_n = V_{cr} \quad (3)$$

where

$$\lambda_v = \sqrt{V_y/V_{cr}}, \quad (4)$$

$$V_y = A_w 0.60 F_y, \quad (5)$$

$V_{cr}$  = critical elastic shear buckling force.

For members with flat webs where  $V_{cr}$  is determined only for the web, these expressions yield the same results as in [29], for more unique cross-section  $V_{cr}$  can be determined by finite element analysis or other methods. Further research to validate these expressions for unique sections is needed.

### 7.2. Inelastic reserve capacity in beams

Inelastic bending capacity exists in cold-formed steel beams, despite their fundamentally thin-walled nature. For example, for the experimental results reported in Figs. 3 and 4, of over 500 flexural tests on cold-formed steel beams approximately 100 tests are found where the bending capacity reaches 95%  $M_y$  or greater including observations as high as 118%  $M_y$ , where  $M_y$  is the moment at first yield. Current methods to account for inelastic reserve capacity, see e.g. [29], are highly involved and restricted in their use. A Direct Strength Method that accounts for inelastic reserve has recently been developed [42].

Using elementary beam mechanics, and assuming elastic–perfectly plastic material, the inelastic compressive strain at failure is back-calculated for the tested members. Simple relationships between local and distortional cross-section slenderness to predict average inelastic strain demands, and a relationship between average strain demand and inelastic bending strength are established. These relationships are combined to provide direct design expressions that connect cross-section slenderness in local or distortional buckling with the inelastic bending strength of cold-formed steel beams. The tested members are also augmented by a detailed finite element study of inelastic local and distortional buckling and the inelastic strains sustained at failure. The elementary mechanics models agree well with the finite element models for the average membrane strains, but peak membrane and flexural strains can be significantly higher. Thus, the local strain demands on the section can be significantly higher than the predicted average inelastic strain demands; nonetheless, predicted strain demands remain lower than expected ductility for commonly used sheet steels.

### 7.3. Members with holes

Research is actively underway to extend the Direct Strength Method to members with holes [30–33,43–47]. (Note, the work in [32,33] is an updated version of [30,31].) The primary complication with extending the Direct Strength Method to members with holes is that the hole introduces the potential for interactive buckling modes triggered by the hole size, spacing, geometry, etc. The finite strip method is not well suited to handle members with holes therefore elastic buckling analysis, the key input in the Direct Strength Method, must at least in the research phase, be completed by general purpose finite element analysis.

In [43–45] data on existing cold-formed steel columns with holes is gathered and eigenvalue elastic buckling analysis is completed using shell element based finite element models that explicitly include the holes and treat the boundary conditions accurately. Model results, such as shown in Fig. 8 where distortional buckling occurs near the hole, but local buckling away from the hole are common. The existing Direct Strength Method expressions, but with  $P_{cr\ell}$ ,  $P_{crd}$ , and  $P_{cre}$  defined as the minimum elastic buckling mode that displays characteristics of local, distortional, and global buckling respectively, were found to provide a reasonable and conservative strength prediction.

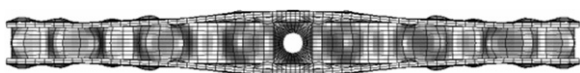


Fig. 8. Mixed local and distortional mode that occurs because of a hole in a C-section column.

Work on columns with holes continues with (i) analysis to determine the influence of hole spacing, (ii) new experiments on columns with holes to augment the data for sections failing with high potential for distortional failures and/or local–distortional interaction, and (iii) nonlinear finite element collapse simulations to further augment the existing and new experimental results [46,47]. In addition, a parallel study on beams with holes has also initiated and initially shows that with proper care in determining elastic buckling the existing Direct Strength Method provisions appear adequate for beams as well [47].

#### 7.4. Angles

Although angles are geometrically one of the simplest cold-formed steel members they are not prequalified for use in the Direct Strength Method implementation in [2]. Recently, Rasmussen in [48] extended his work on angles to include a Direct Strength Method approach. The work explicitly considers eccentricity — thus requiring a beam–column approach even for nominally concentrically loaded angle columns. Consistent with the Direct Strength Method the developed beam–column approach uses the stability of the angle under the applied compression + bending stresses which accurately reflects the fact that some eccentricities (away from the legs) benefit the strength and others (towards the legs) do not.

Work performed in [49] examines a Direct Strength Method approach that ignores eccentricity for angle columns, and also further explores the relationship between local-plate buckling and global-torsional buckling of equal leg angle columns; these authors argue that when one considers the potential for multiple half-waves along the length local-plate and global-torsional should be treated as unique modes. For now, the Direct Strength Method detailed in [48] is the most consistent and rational extension of current design methodologies, though the work in [49] may eventually provide a simpler approach.

#### 7.5. Beam–columns

The design of beam–columns represents an opportunity for the Direct Strength Method to significantly diverge from current practice. Since the stability of the section can be considered directly under the applied loads ( $P$ ) and moments ( $M$ ) the interaction between  $P$  and  $M$  becomes cross-section specific; instead of the invariant interaction equations used in design specifications such as [29]. A basic methodology for the application of the Direct Strength Method for beam columns was proposed in [50,51] and a complete design example using this methodology provided in [37]. The method is conceptually summarized in Fig. 9 — where a cross-section specific interaction diagram is constructed for the sections

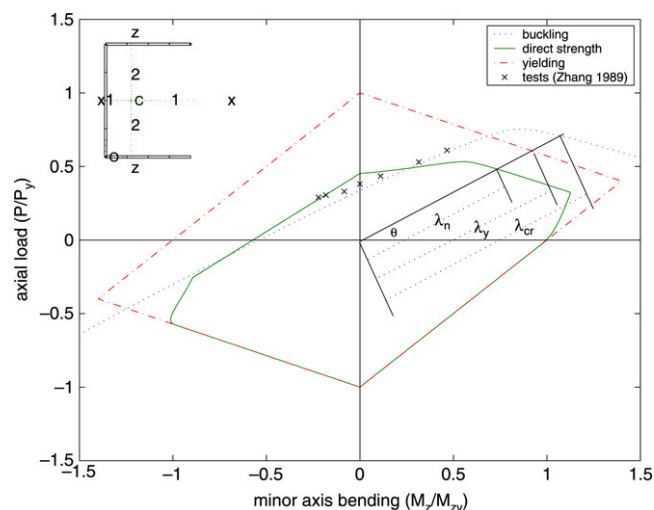


Fig. 9. Proposed interaction diagram solutions for local buckling of unlippped C.

reported in [52] and discussed in [53]. Please note, the results of Fig. 9 differ from those reported in [53], in which it was assumed that a linear interaction diagram could be used for the Direct Strength Method, and no elastic buckling analysis was performed for the eccentric loading.

For any applied combination of  $P$  and  $M$  (which defines the angle  $\theta$  in the interaction diagram) the combination that causes first yield,  $\lambda_y$ , and elastic buckling,  $\lambda_{cr}$ , (typically determined by finite strip analysis) are constructed. Using the same basic Direct Strength Method equations as before, but now replacing, e.g.  $P_{cr}$  and  $P_y$  with  $\lambda_{cr}$  and  $\lambda_y$  — the nominal capacity,  $\lambda_n$ , may be determined. An example of the resulting Direct Strength Method interaction curve is illustrated in Fig. 9. As discussed above, the methodology has been applied to angles in [48]. Comparison to long-column data is provided in [54] with further discussion and an example in [55]. Further experimental and analytical research in this area is currently underway.

#### 7.6. Using pure mode analysis from GBT or cFSM

Application of the Direct Strength Method is greatly aided by computational elastic buckling analysis. In fact, the development of the Direct Strength Method equations relied on the finite strip method, in particular [10]. However, the finite strip method does not always provide a definitive identification of the modes (i.e. which result is local, distortional, and/or global buckling), see [56] for example. Further, the finite element method (using plate or shell elements to comprise the section) provides no definitive method for identifying the modes. The Direct Strength Method requires that the modes be positively identified so that the equations may be applied. Generalized Beam Theory (GBT) [57,58], and now the constrained Finite Strip Method (cFSM) [10,59,60] provide methods for definitively separating the buckling modes from one another. This not only provides the potential for a cleaner and clearer implementation of the Direct Strength Method, but goes much further to opening up the possibility of automating the strength calculation, which enables optimization efforts, such as [61].

One word of caution about the application of the pure mode solutions of GBT (e.g. [62,63]) or cFSM, they are not identically the same as those used in developing the Direct Strength Method. As shown in [64,65] the minima in the finite strip method curve (e.g., Fig. 1(b)) include interaction with the other modes. In the case of local and global buckling this interaction generally is small, but in the case of distortional buckling the minima (i.e.  $P_{crd}$ ) identified by the conventional finite strip method may be as much as 10% or more lower than that identified by GBT or cFSM when only focused on distortional buckling. While it may be possible to recalibrate the Direct Strength Method curves to these “pure mode” solutions for now it is recommended that the GBT or cFSM solutions be used only for determining the critical half-wavelength but the “all mode” or conventional finite strip method solution be used for determining the elastic buckling load (or moment).

7.7. Other materials: Stainless steel, hot-rolled steel, aluminum, plastics

While not the focus of this review, the application of the Direct Strength Method to other materials where cross-section stability plays an important or dominant role in the strength determination is underway. For example, in stainless steel see [66], for hot-rolled steel see [55], for aluminum see [67–69], and for thermoplastics see [70]. The basic methodology has even proved useful in investigating the stability of more unique cross-sections such as the human femur [71].

7.8. Elevated temperatures

Researchers [72,73] have begun to investigate the applicability of the Direct Strength Method for the design of cold-formed steel members under fire conditions. The work is in its beginning stages and is numerical in nature. Using shell element based finite element models and appropriately modifying  $E$  and  $f_y$  to reflect a simulated elevated temperature both research groups show good agreement with the Direct Strength Method expressions (suitably modified for the lower  $E$  and  $f_y$ ). Significant research in this area remains, but the initial results are promising.

8. Conclusions

The Direct Strength Method is a new design methodology for cold-formed steel members. The method has been formally adopted as an alternative design procedure in Appendix A of the North American Specifications for the Design of Cold-Formed Steel Structural Members, as well as in the Australian/New Zealand Standard for cold-formed steel design. The Direct Strength Method employs gross cross-section properties, but requires an accurate calculation of member elastic buckling behaviour. Numerical methods, such as the finite strip method or generalized beam theory, are the best choice for the required stability calculations. The reliability of the Direct Strength Method equals or betters the traditional Effective Width Method for a large database of tested beams and columns. Extensive

design aids are now available for engineers who want to apply the Direct Strength Method in design. Expansion of the Direct Strength Method to cover, shear, inelastic reserve, and members with holes are all underway. In addition, development of a Direct Strength Method for beam–columns continues and will provide cross-section specific interaction with far greater accuracy than the simple (essentially linear) interaction equations in current use. Much work remains for the continued development of the Direct Strength Method, but the efforts of many research groups around the world makes it clear that the Direct Strength Method is on path to be a completely viable alternative design procedure for cold-formed steel member design.

Acknowledgments

The American Iron and Steel Institute is gratefully acknowledged for their support in nearly all of the research presented herein. In addition, the author would like to acknowledge the National Science Foundation under Grant No. CMS-0448707 for their funding support. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation. Finally, recent research by Tom Sputo and Jennifer Tovar that was shared with the author lead to the inclusion of Fig. 5.

Appendix A. Direct Strength Method for columns

(As excerpted from Appendix 1 of the North American Specification for the Design of Cold-Formed Steel Structural Members, 2004 Supplement to the 2001 Edition.)

1.2.1. Column design

The nominal axial strength,  $P_n$ , is the minimum of  $P_{ne}$ ,  $P_{n\lambda}$ , and  $P_{nd}$  as given below. For columns meeting the geometrical and material criteria of Section 1.1.1.1,  $\Omega_c$  and  $\phi_c$  are as follows:

USA and Mexico		Canada
$\Omega_c$ (ASD) 1.80	$\phi_c$ (LRFD) 0.85	$\phi_c$ (LSD) 0.80

For all other columns,  $\Omega$  and  $\phi$  of Section A1.1(b) apply.

1.2.1.1. Flexural, torsional, or torsional–flexural buckling

The nominal axial strength,  $P_{ne}$ , for flexural, ... or torsional–flexural buckling is

$$\text{for } \lambda_c \leq 1.5 P_{ne} = \left(0.658 \lambda_c^2\right) P_y \tag{1.2.1.1}$$

$$\text{for } \lambda_c > 1.5 P_{ne} = \left(\frac{0.877}{\lambda_c^2}\right) P_y \tag{1.2.1.2}$$

where

$$\lambda_c = \sqrt{P_y / P_{cre}} \tag{1.2.1.3}$$

$$P_y = A_g F_y \tag{1.2.1.4}$$

$P_{cre}$  = Minimum of the critical elastic column buckling load in flexural, torsional, or torsional–flexural buckling ...

1.2.1.2. Local buckling

The nominal axial strength,  $P_{n\lambda}$ , for local buckling is

$$\text{for } \lambda_\ell \leq 0.776 \quad P_{n\lambda} = P_{ne} \quad (1.2.1.5)$$

for  $\lambda_\ell > 0.776$

$$P_{n\lambda} = \left[ 1 - 0.15 \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} \right] \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} P_{ne} \quad (1.2.1.6)$$

where

$$\lambda_\ell = \sqrt{P_{ne}/P_{cr\ell}} \quad (1.2.1.7)$$

$P_{cr\ell}$  = Critical elastic local column buckling load ...

$P_{ne}$  is defined in Section 1.2.1.1.

1.2.1.3. Distortional buckling

The nominal axial strength,  $P_{nd}$ , for distortional buckling is

$$\text{for } \lambda_d \leq 0.561 \quad P_{nd} = P_y \quad (1.2.1.8)$$

for  $\lambda_d > 0.561$

$$P_{nd} = \left( 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \left( \frac{P_{crd}}{P_y} \right)^{0.6} P_y \quad (1.2.1.9)$$

where

$$\lambda_d = \sqrt{P_y/P_{crd}} \quad (1.2.1.10)$$

$P_{crd}$  = Critical elastic distortional column buckling load ...

$P_y$  is given in Eq. (1.2.1.4).

**Appendix B. Direct Strength Method for beams**

(As excerpted from Appendix 1 of the North American Specification for the Design of Cold-Formed Steel Structural Members, 2004 Supplement to the 2001 Edition.)

1.2.2. Beam design

The nominal flexural strength,  $M_n$ , is the minimum of  $M_{ne}$ ,  $M_{n\lambda}$ , and  $M_{nd}$  as given below. For beams meeting the geometrical and material criteria of Section 1.1.1.2,  $\Omega_b$  and  $\phi_b$  are as follows:

USA and Mexico		Canada
$\Omega_c$ (ASD)	$\phi_c$ (LRFD)	$\phi_c$ (LSD)
1.67	0.90	0.85

For all other beams,  $\Omega$  and  $\phi$  of Section A1.1(b) apply.

1.2.2.1. Lateral-torsional buckling

The nominal flexural strength,  $M_{ne}$ , for lateral-torsional buckling is

$$\text{for } M_{cre} < 0.56M_y \quad M_{ne} = M_{cre} \quad (1.2.2.1)$$

for  $2.78M_y \geq M_{cre} \geq 0.56M_y$

$$M_{ne} = \frac{10}{9} M_y \left( 1 - \frac{10M_y}{36M_{cre}} \right) \quad (1.2.2.2)$$

$$\text{for } M_{cre} > 2.78M_y \quad M_{ne} = M_y \quad (1.2.2.3)$$

where

$$M_y = S_f F_y, \quad \text{where } S_f \text{ is the gross section modulus} \\ \text{referenced to the extreme fibre in first yield} \quad (1.2.2.4)$$

$M_{cre}$  = Critical elastic lateral-torsional buckling moment ...

1.2.2.2. Local buckling

The nominal flexural strength,  $M_{n\lambda}$ , for local buckling is

$$\text{for } \lambda_\ell \leq 0.776 \quad M_{n\lambda} = M_{ne} \quad (1.2.2.5)$$

for  $\lambda_\ell > 0.776$

$$M_{n\lambda} = \left( 1 - 0.15 \left( \frac{M_{cr\ell}}{M_{ne}} \right)^{0.4} \right) \left( \frac{M_{cr\ell}}{M_{ne}} \right)^{0.4} M_{ne} \quad (1.2.2.6)$$

where

$$\lambda_\ell = \sqrt{M_{ne}/M_{cr\ell}} \quad (1.2.2.7)$$

$M_{cr\ell}$  = Critical elastic local buckling moment ...

$M_{ne}$  is defined in Section 1.2.2.1.

1.2.2.3. Distortional buckling

The nominal flexural strength,  $M_{nd}$ , for distortional buckling is

$$\text{for } \lambda_d \leq 0.673 \quad M_{nd} = M_y \quad (1.2.2.8)$$

for  $\lambda_d > 0.673$

$$M_{nd} = \left( 1 - 0.22 \left( \frac{M_{crd}}{M_y} \right)^{0.5} \right) \left( \frac{M_{crd}}{M_y} \right)^{0.5} M_y \quad (1.2.2.9)$$

where

$$\lambda_d = \sqrt{M_y/M_{crd}} \quad (1.2.2.10)$$

$M_{crd}$  = Critical elastic distortional buckling moment ...

$M_y$  is given in Eq. (1.2.2.4).

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